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Schwarz algorithms for ocean-atmosphere coupled problems including turbulent boundary layer parameterizations

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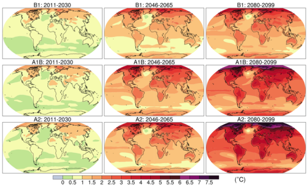
DD26, December 9th 2020

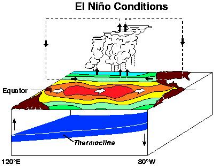
PhD under the supervision of Eric Blayo and Florian Lemarié

Application of ocean-atmosphere coupling

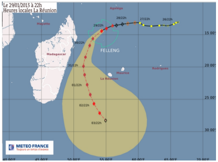
Various physical phenomena are governed by the ocean-atmosphere coupling: long term predictions to short term predictions.



climate modeling



seasonal forecasts

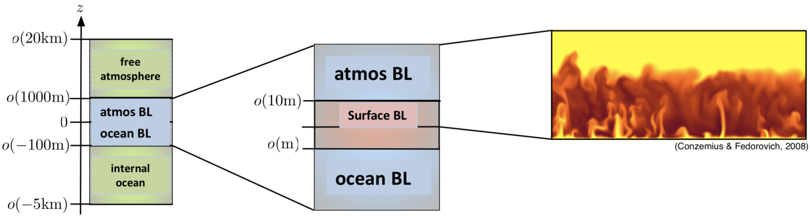


short term predictions

Improve the representation of ocean-atmosphere interactions

Complexity of ocean-atmosphere fluxes

- Turbulente Boundary layer → complex parametrization.

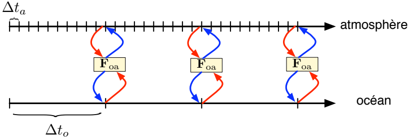


- Near the interface : fluxes estimated by (complicated) formulas depending on the jump of the solution.

The ocean-atmosphere coupling algorithms

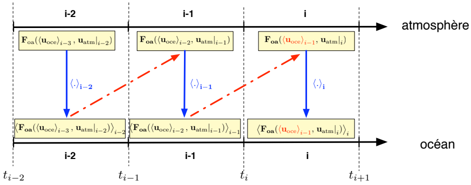
Two current approaches, both mathematically unsatisfactory:

Synchronous coupling at the time step (local in time)



- a lot of communication $\Rightarrow$ inefficient implementations
- physical validity and numerical stability issues **Lemarié & al.** (2015), **Beljaars & al.** (2017)

Asynchronous coupling by time windows (global in time)



- balance of the average flows over each time window
- synchronization problem

Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

Objectives : improve the mathematical coupling methods

Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.
Objectives : improve the mathematical coupling methods

A numerical method that would solve these problems
$\Rightarrow$ Schwarz algorithms

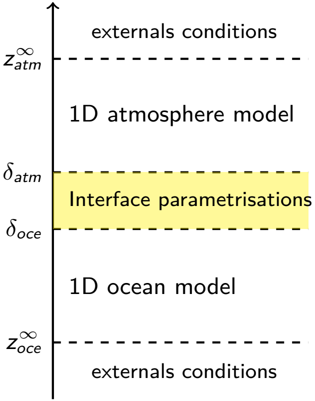
French COCOA ANR Project: Study of an iterative process on ocean-atmosphere coupling. In particular

- Implementation of Schwarz algorithms in realistic climate models
- Theoretical work on these algorithms in this context.

A 1D Simplified coupled ocean-atmosphere model

Hypotheses :

- Focus on the dynamical part
- Physical restriction (1D)
- Taking into account turbulent parametrisations



A reasonably realistic model on $\mathbf{U} = (u, v)^T$ horizontal ocean/atmosphere currents.

Interface is a buffer zone with its own parameterization.

Non-linear coupled ocean-atmosphere model

+ external conditions

z_{atm}^{∞}

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{atm}(u^*, z)\partial_z \mathbf{U}) = \mathcal{F}$$

δ_{atm}

$$\textbf{SBL}\,(\|\mathbf{U}(\delta_{atm}) - \mathbf{U}(\delta_{oce})\|) \rightarrow u^*$$

$$\rho_a \nu_{atm}(u^*, \delta_{atm})\partial_z \mathbf{U}(\delta_{atm}) = \rho_a (u^*)^2 \mathbf{e}_\tau = \rho_o \nu_{oce}(u^*, \delta_{oce})\partial_z \mathbf{U}(\delta_{oce})$$

δ_{oce}

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{oce}(u^*, z)\partial_z \mathbf{U}) = \mathcal{F}$$

z_{oce}^{∞}

+ external conditions

ocean atmosphere specificities :

- Coriolis effect
- non constants viscosities
- non linear equation
- non linear interface condition

Linear coupled ocean-atmosphere model

+ Dirichlet conditions

z_{atm}^{∞}

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{atm}(z)\partial_z \mathbf{U}) = \mathcal{F}$$

$$\rho_a \nu_{atm}(0^+)\partial_z \mathbf{U}(0^+) = \rho_o \nu_{oce}(0^-)\partial_z \mathbf{U}(0^-)$$

0

$$\mathbf{U}(0^-) = \mathbf{U}(0^+)$$

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{oce}(z)\partial_z \mathbf{U}) = \mathcal{F}$$

z_{oce}^{∞}

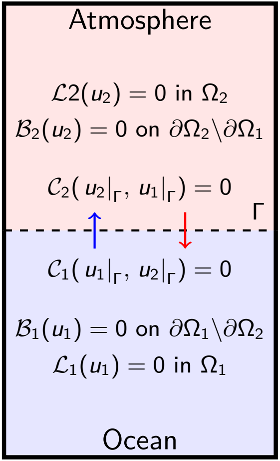
+ Dirichlet conditions

$\Rightarrow$ Ekman Problem widely used by physicists: **Ekman** (1905), **Madsen** (1977), **Grisogono** (1995), **Lewis & Belcher** (2004)

Linear problem specificities :

- Coriolis effect
- non constants viscosities

Schwarz algorithm



first guess u_2^0 then

$$\begin{cases} \mathcal{L}_1 u_1^n = \mathcal{F}_1 & \text{on } \Omega_1 \times]0, T[\\ \mathcal{B}_1 u_1^n = \mathcal{G}_1 & \text{on } \partial\Omega_1^{ext} \times]0, T[\\ u_1^n(t=0) = u_0 & \text{on } \Omega_1 \\ \mathcal{C}_{1,1} u_1^n = \mathcal{C}_{1,2} u_2^{n-1} & \text{on } \Gamma \end{cases}$$

$$\begin{cases} \mathcal{L}_1 u_2^n = \mathcal{F}_2 & \text{on } \Omega_2 \times]0, T[\\ \mathcal{B}_2 u_2^n = \mathcal{G}_2 & \text{on } \partial\Omega_2^{ext} \times]0, T[\\ u_2^n(t=0) = u_0 & \text{on } \Omega_2 \\ \mathcal{C}_{2,2} u_2^n = \mathcal{C}_{2,1} u_1^n & \text{on } \Gamma \end{cases}$$

Convergence factor for linear problems

$$\rho^{obs} = \frac{||e_j^n(z=0)||_2}{||e_j^{n-1}(z=0)||_2} \qquad e_j^n = u_j^n - u^{exact}$$

- 1D Stationary case : solve the equation for each iteration
- 1D Non-stationary case : use Fourier transform in time

$$\Rightarrow \rho(\omega) = \frac{\left|\widehat{e_j^n}(\omega,0)\right|}{\left|\widehat{e_j^{n-1}}(\omega,0)\right|}$$

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$$\min_{\omega_{\min} \leq |\omega| \leq \omega_{\max}} \rho(\omega) \leq \rho^{obs} \leq \max_{\omega_{\min} \leq |\omega| \leq \omega_{\max}} \rho(\omega)$$

Discretized-in-time algorithm: $\omega_{max} = \dfrac{\pi}{\Delta t}$ and $\omega_{\min} = \dfrac{\pi}{T}$

Coupled Ekman problems

+ Dirichlet conditions

z_{atm}^{∞}

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{atm}(z)\partial_z \mathbf{U}) = \mathcal{F}$$

$$\rho_a \nu_{atm}(0^+)\partial_z \mathbf{U}(0^+) = \rho_o \nu_{oce}(0^-)\partial_z \mathbf{U}(0^-)$$

0

$$\mathbf{U}(0^-) = \mathbf{U}(0^+)$$

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{oce}(z)\partial_z \mathbf{U}) = \mathcal{F}$$

z_{oce}^{∞}

+ Dirichlet conditions

$\Rightarrow$ Specificities :

- Coriolis effect ($\mathbf{U}$ components are coupled)
- Non constant diffusion coefficients with interface discontinuity

State of the art

		Constante diffusion	Variable in space diffusion
Stationary		*adv-diff 2D* Japhet et al., 2001 *eq. Helmholtz* Dubois, 2007 Magoulès et al., 2004	*eq. de diffusion* Lions, 1990
Nonstationary	without Coriolis	*heat equation* Gander and Halpern, 2003 *reaction-reaction-diff 2D* Bennequin et al., 2016 and Gander et al., 2007	*diffusion 1D eq.* Lemarié et al., 2013
	with Coriolis	*2D shallow water* Martin, 2003 *primitives eq. 3D* Audusse et al., 2009	*diffusion 1D eq. +* *Coriolis* Thery et al., 2020

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components

$$\begin{cases} \partial_t \mathbf{U}_j + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U}_j - \nu_j \partial_z^2 \mathbf{U}_j = \mathcal{F}_j \\ \quad + \text{Dirichlet external conditions} \\ \quad + \text{Initial conditions} \\ \mathbf{U}_1(0^-) = \mathbf{U}_2(0^+) \\ \nu_1(0^-)\partial_z \mathbf{U}_1 = \nu_2(0^+)\partial_z \mathbf{U}_2 \end{cases}$$

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.

$$\begin{cases} \partial_t \varphi_j + i f \varphi_j - \nu_j \partial_z^2 \varphi_j = \mathcal{F}_{\varphi_j} \\ \\ + \text{ Dirichlet external conditions} \\ + \text{ Initial conditions} \\ \varphi_1(0^-) = \varphi_2(0^+) \\ \nu_1(0^-)\partial_z \varphi_1 = \nu_2(0^+)\partial_z \varphi_2 \end{cases}$$

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error e_j

$$
\begin{cases}
\partial_t e_j + i f e_j - \nu_j \partial_z^2 e_j = 0 \\
+ \text{ Dirichlet external conditions} \\
+ \text{ Null initial condition} \\
e_1(0^-) = e_2(0^+) \\
\nu_1 \partial_z e_1 = \nu_2 \partial_z e_2
\end{cases}
$$

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error e_j
- Fourier Transform $\Rightarrow$ frequencies shifted by f :

$$\begin{cases} i(\omega + f)\widehat{e}_j - \nu_j \partial_z^2 \widehat{e}_j = 0 \\ + \text{ Dirichlet external conditions} \\ \widehat{e}_1(0^-) = \widehat{e}_2(0^+) \\ \nu_1 \partial_z \widehat{e}_1 = \nu_2 \partial_z \widehat{e}_2 \end{cases}$$

With Dirichlet-Neumann interface conditions

$$\begin{cases} \hat{e}_1^n(\omega,0) = & \hat{e}_2^{n-1}(\omega,0) \\ \nu_2 \partial_z \hat{e}_2^n(\omega,0) = & \nu_1 \partial_z \hat{e}_1^n(\omega,0) \end{cases}$$

Convergence factor

- Infinite domains:

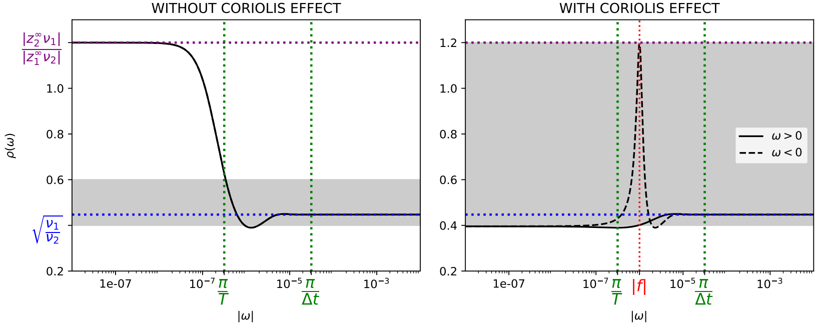
$$\rho_{DN}^{cst}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}}$$

  Independent of time frequency

- Finite domains:

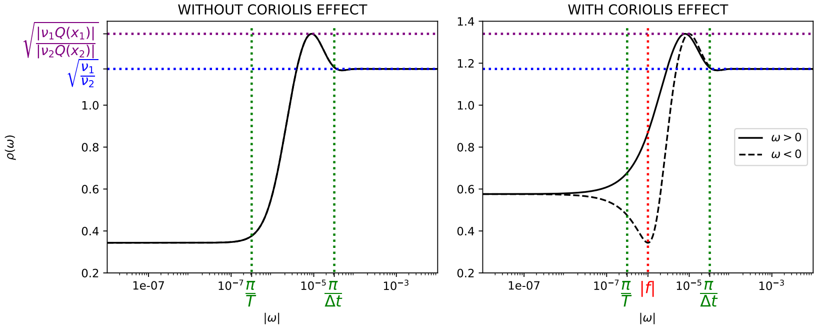
$$\rho_{DN}^{cst}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}} \left| \frac{\tanh\left(z_2^\infty \sqrt{i\,\frac{\omega+f}{\nu_2}} \right)}{\tanh\left(z_1^\infty \sqrt{i\,\frac{\omega+f}{\nu_1}} \right)} \right|$$

Case 1 : $\left|z_2^{\infty}\sqrt{\nu_1}\right| > \left|z_1^{\infty}\sqrt{\nu_2}\right|$



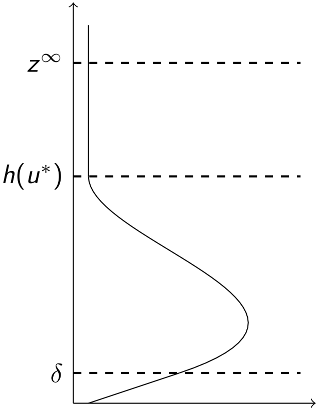
- Convergence factor behavior : $|\rho(\omega)| < \frac{z_2^{\infty}}{z_1^{\infty}}\frac{\nu_1}{\nu_2}$ and $\rho(\omega) \underset{|\omega|\to\infty}{\longrightarrow} \sqrt{\frac{\nu_1}{\nu_2}}$
- Impact of Coriolis : shifts the local maximum and non symmetric graph

Case 2 : $|z_2^\infty \sqrt{\nu_1}| < |z_1^\infty \sqrt{\nu_2}|$



- Convergence factor behavior : $|\rho(\omega)| < \sqrt{\frac{\nu_1}{\nu_2}\frac{Q(x_1)}{Q(x_2)}}$ and $\rho(\omega) \underset{|\omega|\to\infty}{\longrightarrow} \sqrt{\frac{\nu_1}{\nu_2}}$ with $Q(x) = |\tanh((1+i)x)|$ and x_1, x_2 solution of the transcendantale equation
- Impact of Coriolis : shifts the local minimum and non symmetric graph

The effect of turbulence



Parametrisation of turbulence $\Rightarrow$ non constant viscosity

In ocean-atmosphere context :
KPP viscosity (O'Brien, 1970)

- affine profile close to the surface
- parabolic or cubic profile in the turbulent zone
- constant profile in free zone

$\Rightarrow$ convergence for variable viscosity profile

Convergence factor with non-constants viscosities

$$i(f+\omega)\widehat{e}_j(z,t) - \partial_z(\nu_j(z)\partial_z\widehat{e}_j(z,t)) = 0$$

Mathematical tools to calculate converge :

- with $\nu_j(z) = a_j z + b_j$ $\rightarrow$ Bessel's functions
- with $\nu_j(z) = a_j z^2 + b_j z + c_j$ $\rightarrow$ Legendre polynomials.

$\Rightarrow$ The convergence factor depends on the global viscosity profile

Convergence factor with non-constants viscosities

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$\Rightarrow$ The convergence factor depends on the global viscosity profile

Dirichlet-Neumann interface conditions

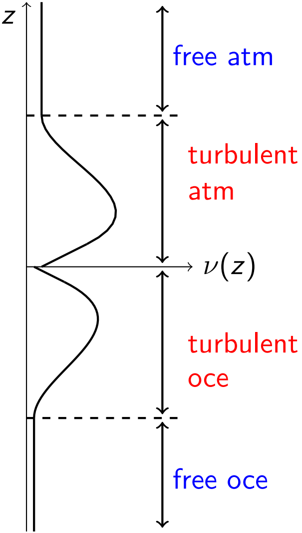
$\rho_{DN}(\omega) \underset{|\omega|\to\infty}{\longrightarrow} \sqrt{\dfrac{\nu_1(0)}{\nu_2(0)}}$ for all viscosities profils

$$\rho_{DN}^{aff}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{\mu_1 \ln(1+1/\mu_1)}$$

$$\rho_{DN}^{par}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{4\mu_1 \arccos(\sqrt{1+4\mu_1})}$$

with $\mu_1 = \left| \dfrac{\nu_1(0)}{\partial_z \nu_1(0) z_1^\infty} \right|$

The particular case ocean-atmosphre couplig



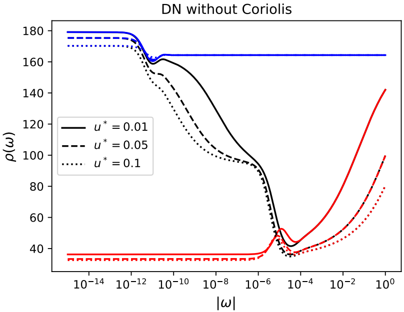
Stationary case for any interface condition :

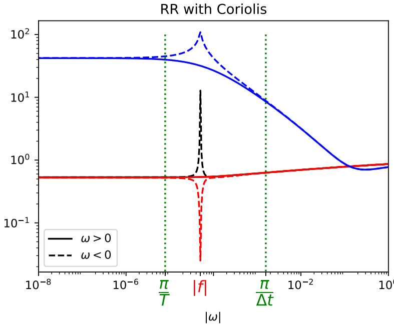
- without Coriolis $\rightarrow$ free zones have a huge influence
- with Coriolis $\rightarrow$ turbulent zones have a bigger influence

The particular case ocean-atmosphere coupling

Non-Stationary case for any interface condition :

- $|\omega + f| < 10^{-11} \rightarrow$ influenced by free zone
- $|\omega + f| > 10^{-5} \rightarrow$ influenced by turbulent zone





Example : convergence for all frequencies except frequencies close to $-f$

Conclusion for linear problems

- Impact of Coriolis effect : shift of the graph
  $\Rightarrow$ perturbation of the algorithm's behavior
- Convergence factor depends of the global viscosities profile
  $\Rightarrow$ mathematicals tools to calculate the convergence factor (Bessel and Legendre functions)
  $\Rightarrow$ simplification must be made with caution

$\Rightarrow$ Results are shown in : *S. Thery, C. Pelletier, F. Lemarié and Blayo E., 2020: Coupling two Ekman layers with a Schwarz algorithm. under review*

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Ocean-atmosphere coupling

The Coriolis effect and the turbulence zones have a big impact on the convergence

The non-linear model

+ Dirichlet conditions

z_{atm}^{∞}

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{atm}(u^*, z)\partial_z \mathbf{U}) = \mathcal{F}$$

$$\nu_{atm}(u^*)\partial_z \mathbf{U} = (u^*)^2 \mathbf{e}_\tau$$

δ_{atm}

$$u^* \propto (\|\mathbf{U}(\delta_{atm}) - \mathbf{U}(\delta_{oce})\|)$$

δ_{oce}

$$\rho_{oce}\nu_{oce}(u^*)\partial_z \mathbf{U} = -\rho_{atm}(u^*)^2 \mathbf{e}_\tau$$

$$\partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z(\nu_{oce}(u^*, z)\partial_z \mathbf{U}) = \mathcal{F}$$

z_{oce}^{∞}

+ Dirichlet conditions

Application of Schwarz algorithms on non-linear model

- Study of the well posedness of the problem
- Study of convergence of the algorithm

Current work on the non-linear model

Stationary case : using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect : fast divergence & free zones have a huge influence
- with Coriolis effect : fast convergence & turbulent zones have a bigger influence

Current work on the non-linear model

Stationary case : using tools from linear problem

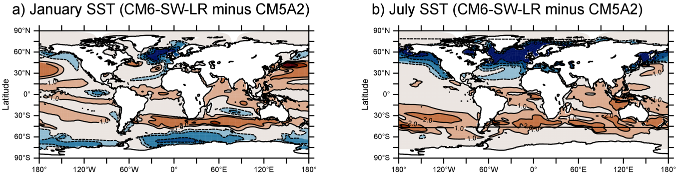
- unique solution consistent with the physical constraints
- without Coriolis effect : fast divergence & free zones have a huge influence
- with Coriolis effect : fast convergence & turbulent zones have a bigger influence

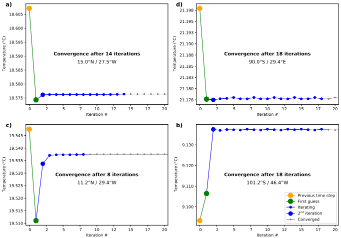
Non stationary case :

No theoretical method for solve this problem
$\Rightarrow$ experimental results $\Rightarrow$ similarities with linear problem behavior :

- unique solution consistent with the physical constraints
- if $\omega_{\text{min}} \leq |f| \leq \omega_{\text{max}} \rightarrow$ divergence
- if $|f| \leq \omega_{\text{min}}$ or $\omega_{\text{max}} \leq |f| \rightarrow$ convergence

Test on a real model





IPSL-CM (3D)

- test on a climate model (3D)
- Convergence in two iterations for 90% points

Marti, O., Nguyen, S., Braconnot, P., Valcke, S., Lemarié, F., and Blayo, E.: A Schwarz iterative method to evaluate ocean-atmosphere coupling schemes. Implementation and diagnostics in IPSL-CM6-SW-VLR, Geosci. Model Dev. Discuss., https://doi.org/10.5194/gmd-2020-307, 2020

Thank you